Non-Linear Image Filtering

Audiovisual Processing CMP-6026A

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Content

- Order Statistics filters
- Morphological filtering
- Morphological operations for object detection
- Morphological operations for edge detection

Order-Statistics Filters

Linear filters compute the sum of products between kernel coefficients and the image neighbourhood.

Instead, replace intensity with a measure obtained by ordering the pixel intensities in the neighbourhood.

- Common filters are **median**, **max** and **min**.

Crop a part of a larger image to uses as an example.

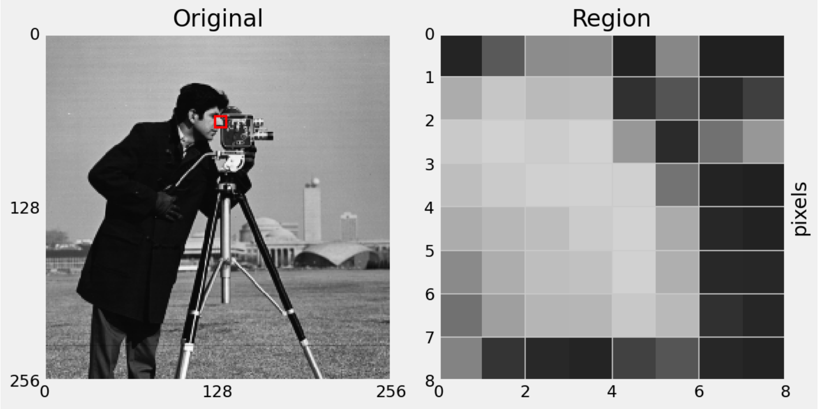


Figure 1: Cameraman crop

First, window a region of size 3x3:

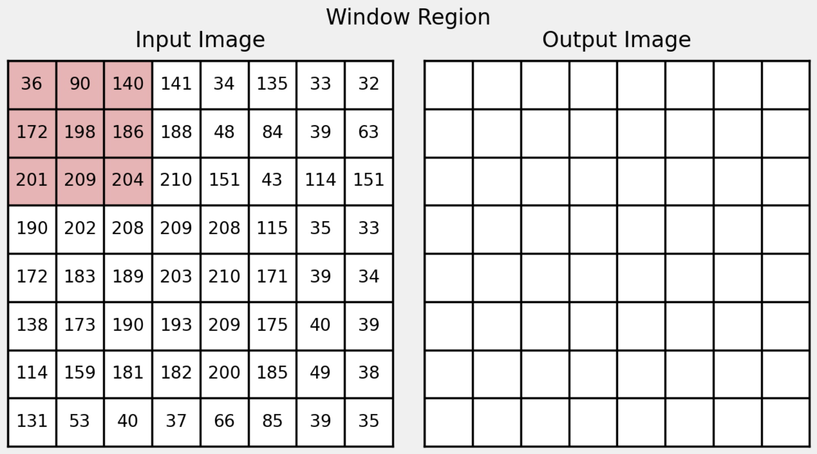


Figure 2: region window

Sort the intensities in the region:

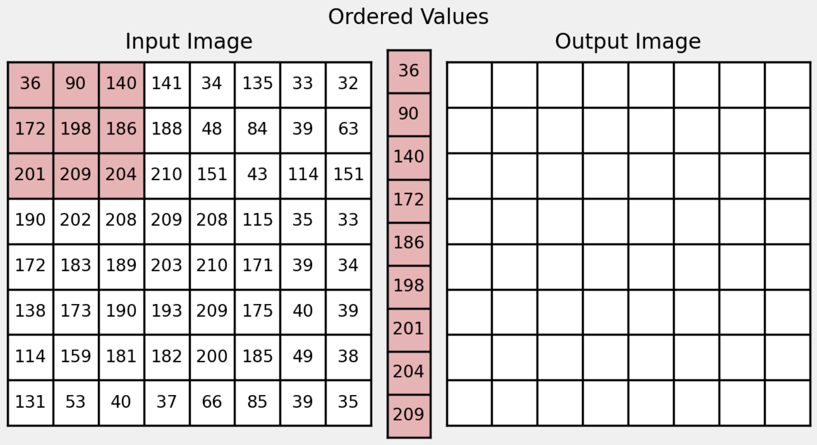


Figure 3: ordered values

Sort the intensities in the region, and select the middle value:

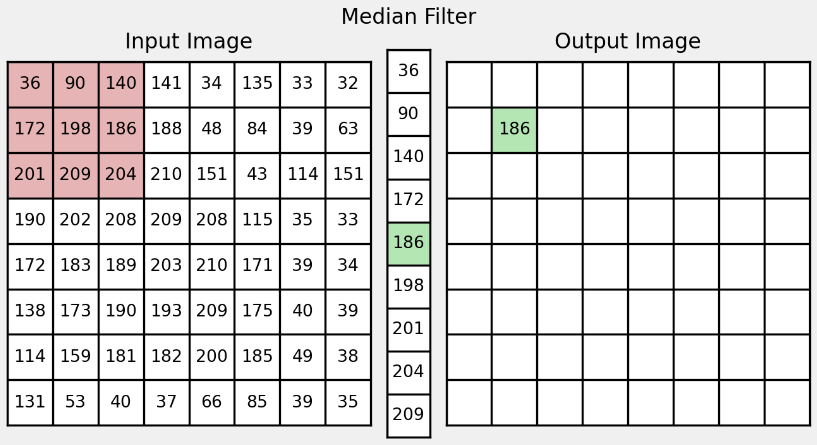


Figure 4: median value

Max filter is similar to median filter, but selects the maximum value:

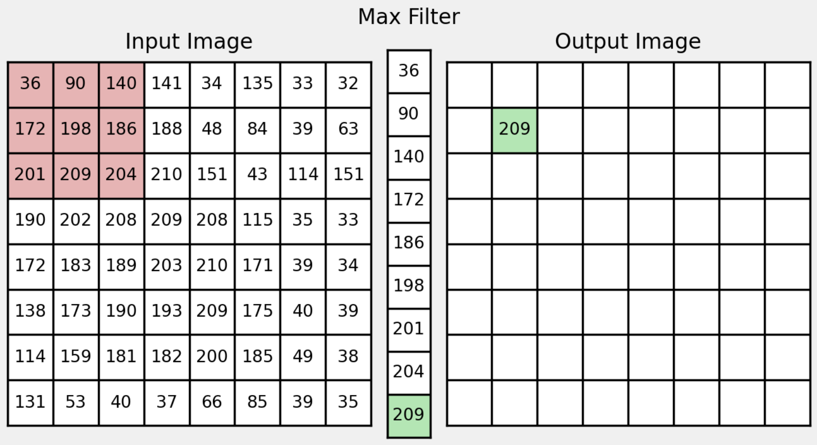


Figure 5: maximum value

Min filter is again similar, but selects the minimum value:

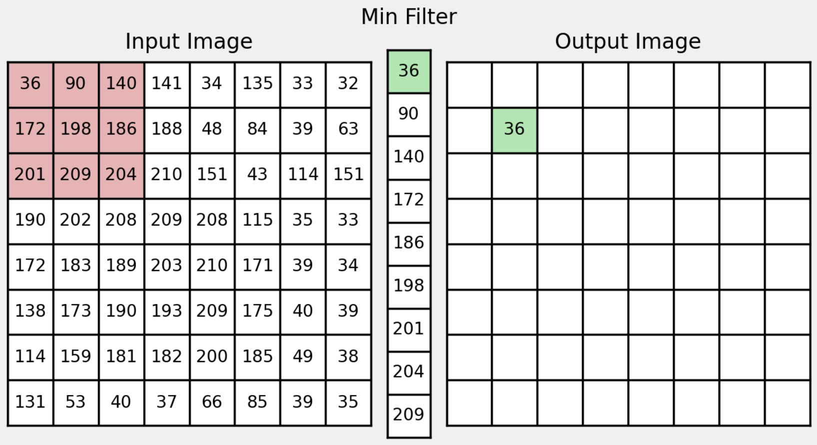


Figure 6: minimum value

Order-statistics filters are good for noise removal where:

- Noise is random
- Noise is not dependent on surrounding pixels

For example, the salt and pepper noise model:

- Noisy pixels are the outliers in the local neighbourhood
- Replace outliers with estimate from image data

Aside: What is salt and pepper noise?

- Also called impulse noise.
- Can be hardware dependent - hot pixels or dead pixels.
- Artificially applied by random selecting a subset of pixels and setting to black or white.

Median Filter

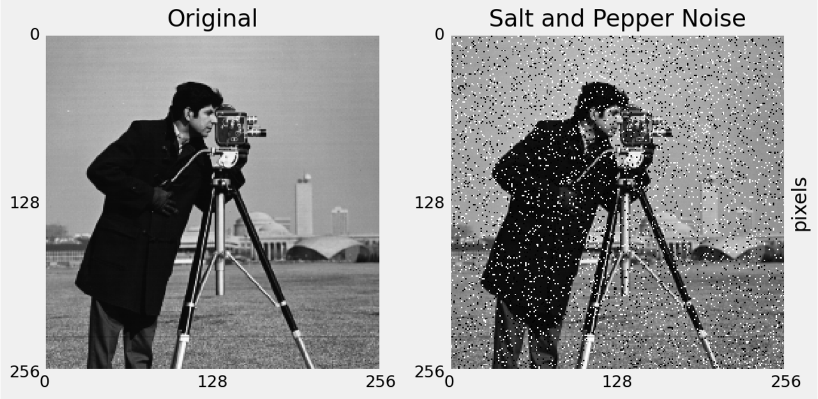


Figure 7: salt and pepper noise, $P = 0.1$

Median Filter

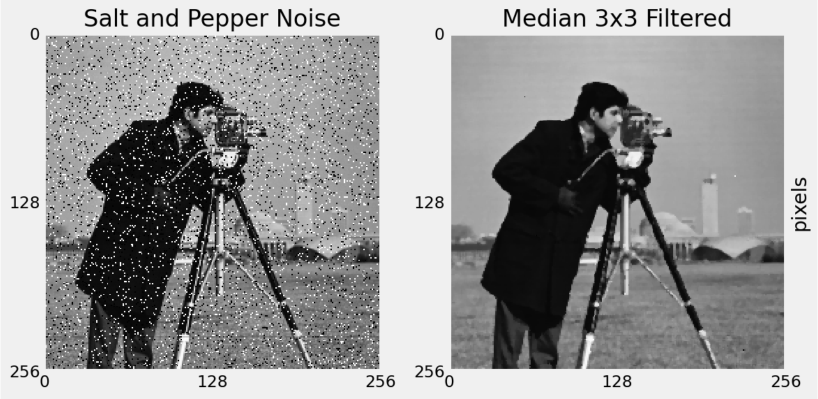


Figure 8: noisy and median filtered images

Median Filter

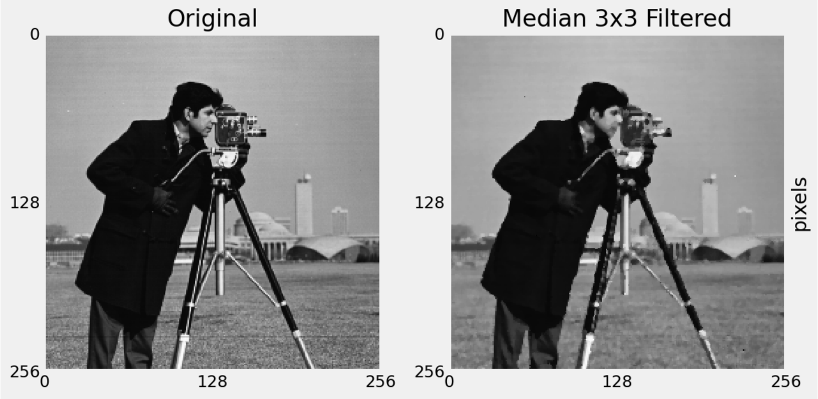


Figure 9: original and median filtered images

Median Filter

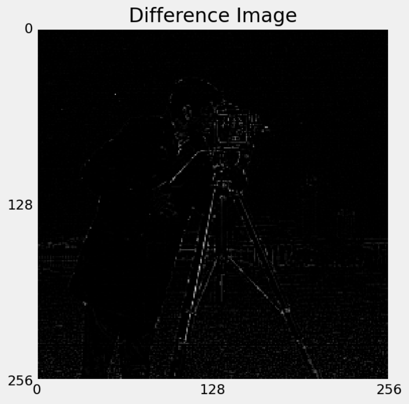


Figure 10: difference image

Median Filter

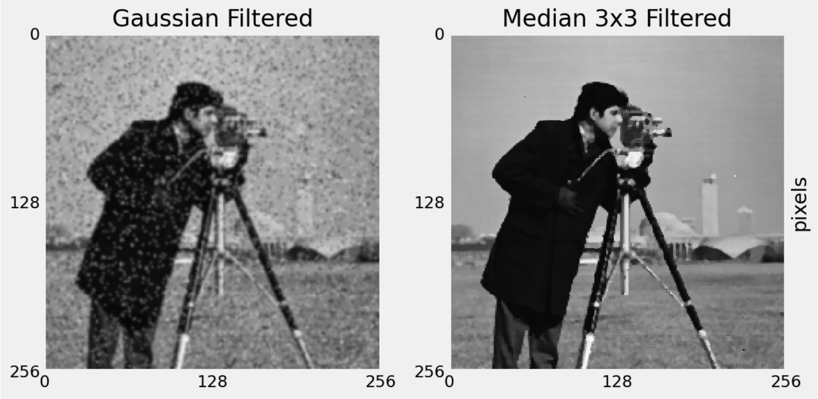
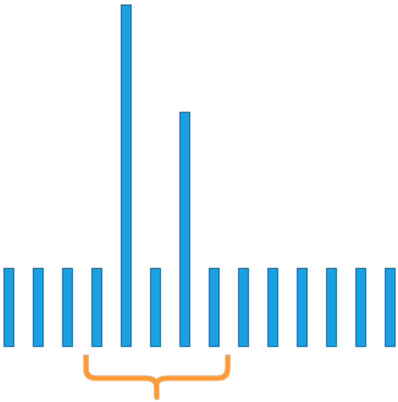


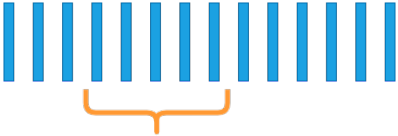
Figure 11: gaussian and median filtered images

Noise removal



If we consider this 5 pixel neighbourhood, what will the median filter do?

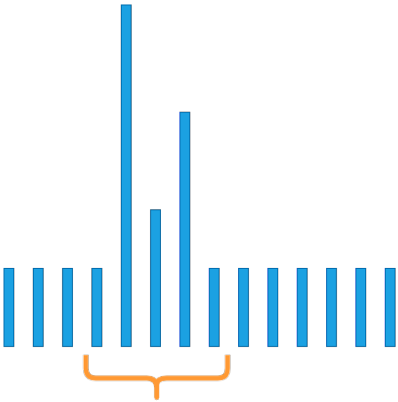
Noise removal



The median filter removes spike noise.

- What will the gaussian filter do?

Noise removal



The Gaussian filter amplifies noise.

Morphological Filters

Operation of the filter is characterised by mathematical morphology.

- Embedded in set theory.
- Useful for thickening and thinning edges and de-noising binary images

Motivation

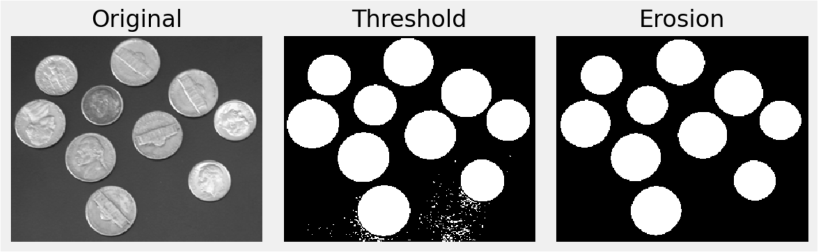


Figure 12: Filtered Threshold Image

Morphological Filters

For binary images:

- White pixels (with intensity 1) can be considered elements in a set.
- Black pixels (with intensity 0) can be considered elements outside of the set.
- Morphological filters are essentially set operations.

Set Notation

Let A be a set in $\mathbb{Z}^2$

the set of *all integers* in 2 dimensions...

Set Notation

$$a \in A \text{ if } a = (x, y) \text{ is an element of set } A$$

this is the *element* symbol...

Set Notation

$$a \notin A \text{ if } a \text{ is not in } A$$

not in...

Set Notation

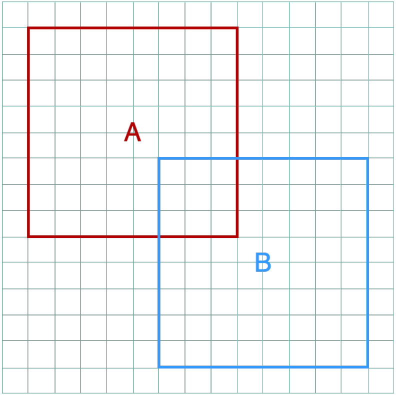
$$C = \{w | w = -d, \ for \ d \in D\}$$

the set of all *w such that*. . .

Set Notation

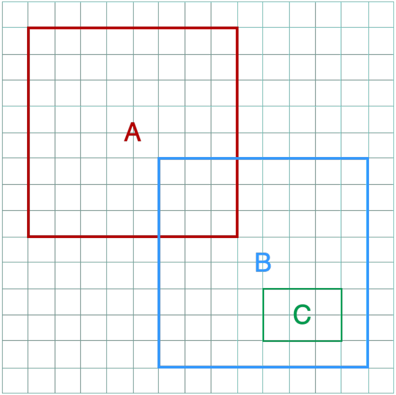
$\emptyset$ is the empty set

Set Operations



Given two sets, A and B, the following can be defined:

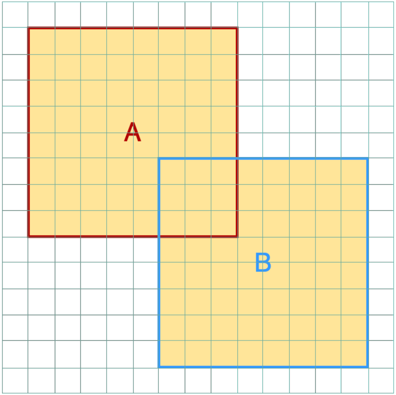
Set Operations



$$C \subseteq B, A \not\subseteq B$$

Subset: a set where all members belong to a given set.

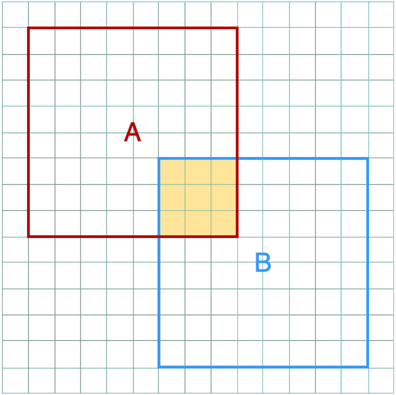
Set Operations



$$A \cup B$$

Union: all elements that are either in set A or set B.

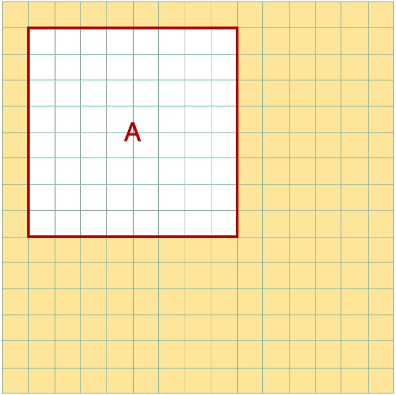
Set Operations



$$A \cap B$$

Intersection: all elements that are common to both A and B.

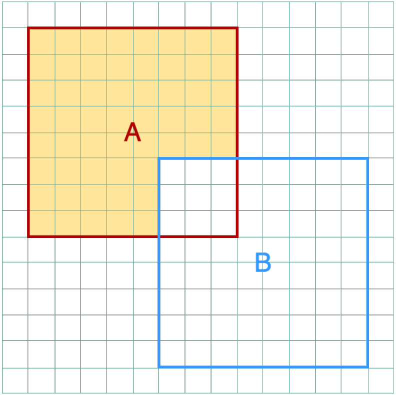
Set Operations



$$A^c\{w|w \notin A\}$$

Complement: the elements not contained in set A

Set Operations



$$A \setminus B = \{w | w \in A, \; w \notin B\}$$

Difference: the elements of set A that are not in set B

Structuring Element

A *binary* image (or mask) that allows us to define neighbourhood structures.

Structuring Element

- Can be different sizes: larger structuring elements produce a more extreme effect.
- Can be different shapes: common to use a disk or cross shape.
- Has a defined origin: usually at the centre.

Structuring Element

1	1	1
1	**1**	1
1	1	1

0	0	1	0	0
0	0	1	0	0
1	1	**1**	1	1
0	0	1	0	0
0	0	1	0	0

0	0	1	0	0
0	1	1	1	0
1	1	**1**	1	1
0	1	1	1	0
0	0	1	0	0

Structuring Element

A structuring element is said to **fit** the image if, for each of its pixels set to 1, the corresponding image pixel is also 1.

The *set* of all displacements such that the image and the structuring element overlap at **every** pixel.

Structuring Element

A structuring element is said to **hit**, an image if, at least for one of its pixels set to 1 the corresponding image pixel is also 1.

The *set* of all displacements such that the image and the structuring element overlap at **any** pixel.

Dilation

$$A \oplus B = \{x, y | B_{x,y} \cap A \neq \emptyset\}$$

Defines dilation of binary image A by structuring element B.

Calculate the binary **OR** of elements in A masked by B.

The structuring element *hits* the image.

Dilation

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	1	1	1	0	0
0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0
0	0	0	1	1	0	0	0	0	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 13: binary image

$A \oplus B$

1	1	1
1	1	1
1	1	1

Figure 14: structured element

Dilation

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	1	1	1	0	0
0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0
0	0	0	1	1	0	0	0	0	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 15: binary image

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0	0	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0
0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0

Figure 16: dilated image

Dilation Example

Figure 17: dilation 5x5

Dilation Example

Figure 18: dilation 15x15

Dilation

- Expands the size of 1-pixel objects
- Smoothes object boundaries
- Closes holes and gaps
- Regions grow

Erosion

$$A \ominus B = \{x, y | B_{x,y} \subseteq A\}$$

Defines erosion of binary image A by structuring element B.

Calculate the binary **AND** of elements in A masked by B.

The structuring element *fits* the image.

Erosion

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	1	1	1	0	0
0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0
0	0	0	1	1	0	0	0	0	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 19: binary image

$$A \ominus B$$

1	1	1
1	1	1
1	1	1

Figure 20: structured element

Erosion

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	1	1	1	0	0
0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0
0	0	0	1	1	0	0	0	0	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 21: binary image

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 22: eroded image

Erosion Example

Figure 23: erosion 5x5

Erosion Example

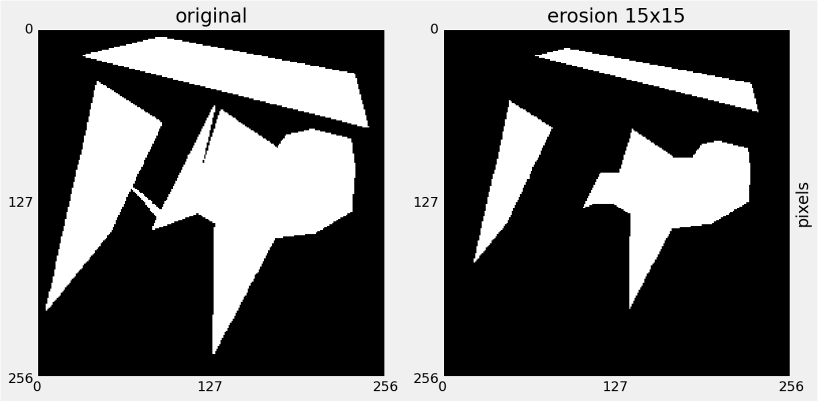


Figure 24: erosion 15x15

Erosion

- Shrinks the size of 1-valued objects.
- Smooths object boundaries.
- Removes peninsulas, fingers, and small objects (such as noise).

Opening

$$A \circ B = (A \ominus B) \oplus B$$

Erosion followed by **Dilation**.

– Has the effect of smoothing contours by breaking narrow connections and eliminating thin protrusions.

Opening Example

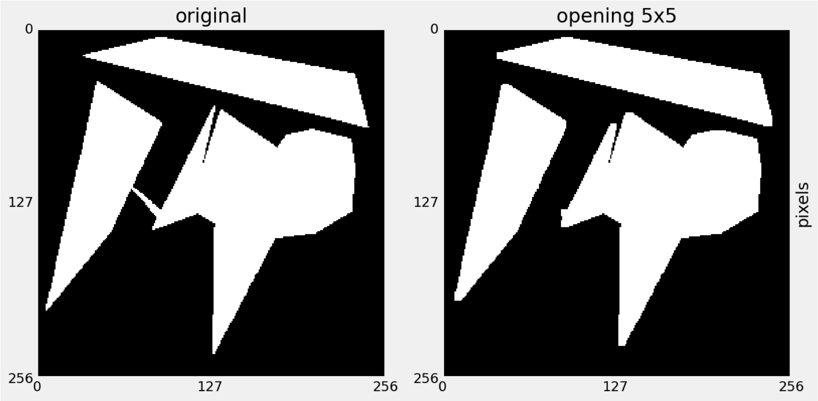


Figure 25: opening 5x5

Opening Example

Figure 26: opening 15x15

Closing

$$A \bullet B = (A \oplus B) \ominus B$$

Dilation followed by **Erosion**.

– Has the effect of smoothing contours by filling narrow gulfs, holes and small gaps.

Closing Example

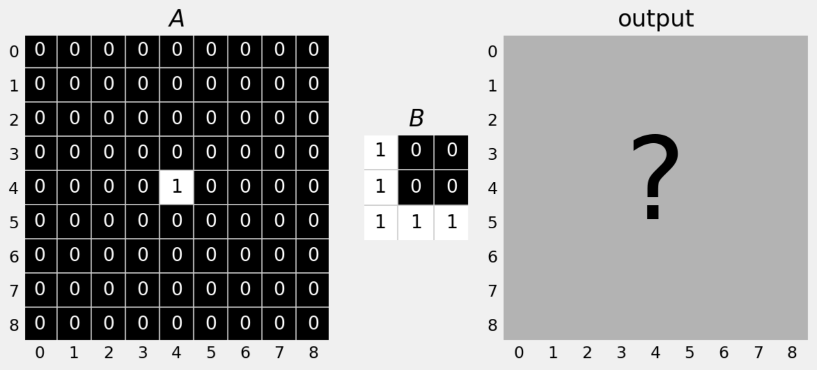
Figure 27: closing 5x5

Closing Example

Figure 28: closing 15x15

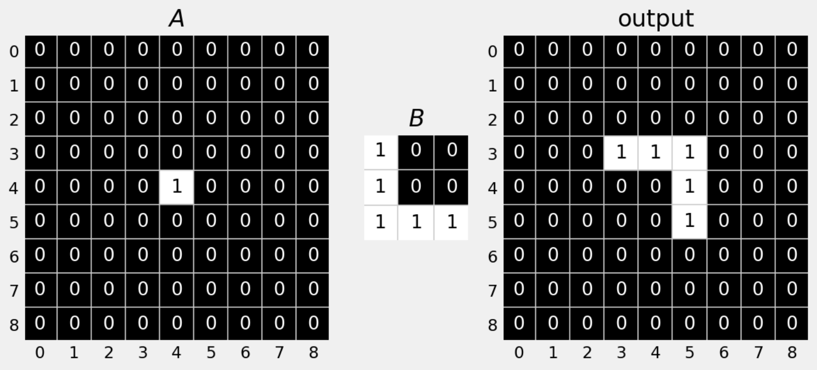
QUESTION?

After performing **dilation** of A by B, what does the resulting binary image look like?



Answer

After performing **dilation** of A by B, what does the resulting binary image look like?



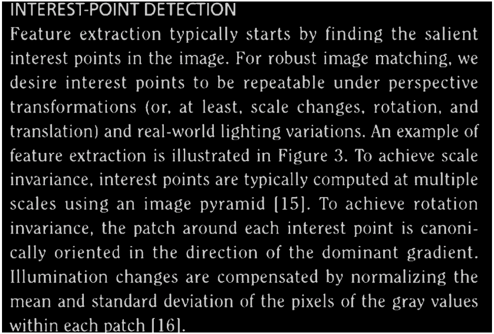
Object Detection

INTEREST-POINT DETECTION

Feature extraction typically starts by finding the salient interest points in the image. For robust image matching, we desire interest points to be repeatable under perspective transformations (or, at least, scale changes, rotation, and translation) and real-world lighting variations. An example of feature extraction is illustrated in Figure 3. To achieve scale invariance, interest points are typically computed at multiple scales using an image pyramid [15]. To achieve rotation invariance, the patch around each interest point is canonically oriented in the direction of the dominant gradient. Illumination changes are compensated by normalizing the mean and standard deviation of the pixels of the gray values within each patch [16].

Figure 29: Original image A

Object Detection



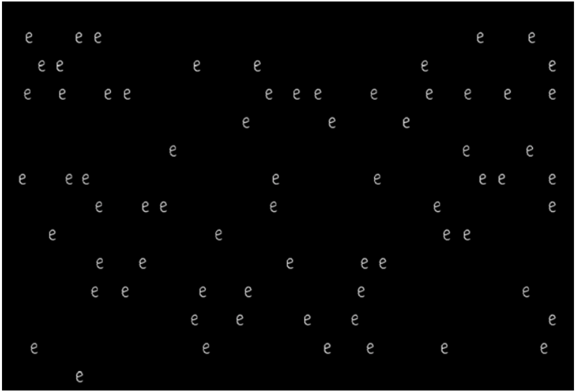
INTEREST-POINT DETECTION

Feature extraction typically starts by finding the salient interest points in the image. For robust image matching, we desire interest points to be repeatable under perspective transformations (or, at least, scale changes, rotation, and translation) and real-world lighting variations. An example of feature extraction is illustrated in Figure 3. To achieve scale invariance, interest points are typically computed at multiple scales using an image pyramid [15]. To achieve rotation invariance, the patch around each interest point is canonically oriented in the direction of the dominant gradient. Illumination changes are compensated by normalizing the mean and standard deviation of the pixels of the gray values within each patch [16].

Figure 30: *A*

Figure 31: *B*

$A \circ B$



Edge Detection

Using difference images, we can detect edges.

- Dilation of A by structural element B, followed by difference with A.
- Erosion of A by B, followed by difference with A.
- Dilation of A by B, followed by difference with erosion of A by B.

Edge Detection

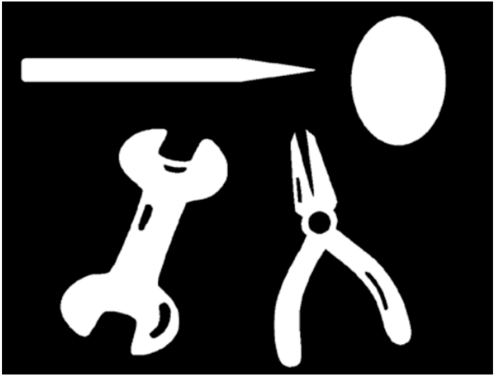
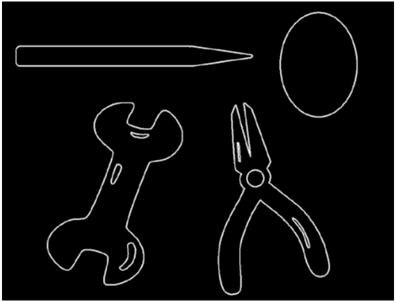


Figure 32: binary A

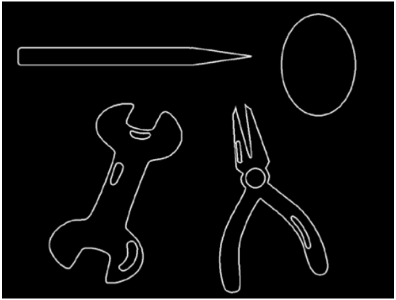
Edge Detection



$$(A \oplus B) - A$$

Dilation Difference image.

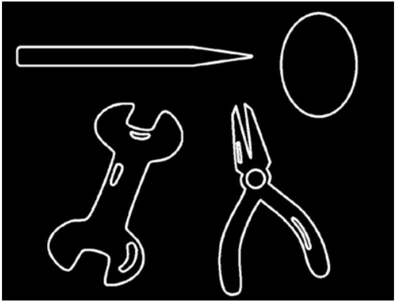
Edge Detection



$$(A \ominus B) - A$$

Erosion Difference image.

Edge Detection



$$(A \oplus B) - (A \ominus B)$$

Dilation Erosion Difference.

Further Reading

Digital Image Processing

- Rafael C. Gonzalez
- Richard E. Woods

Summary

- Order Statistics filters
- Morphological filtering
- Morphological operations for object detection
- Morphological operations for edge detection