Camera Calibration

Computer Vision CMP-6035B

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Contents

Zhang's Method

A method of finding the **intrinsic** parameters of a camera.

- Zhang, Z., 2000. A flexible new technique for camera calibration. IEEE Transactions on pattern analysis and machine intelligence, 22(11), pp.1330-1334.

Point mapping

$$x = PX$$

pixel coordinate — trans-formation — world coordinate

Figure 1: point mapping

Point mapping

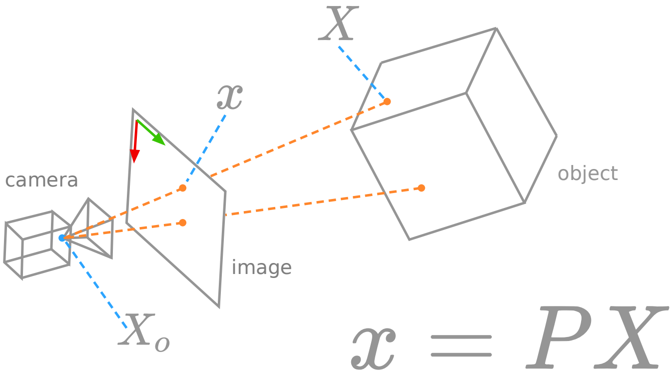

Figure 2: Point to pixel

Direct Linear Transformation

Compute the 11 *intrinsic* **and** *extrinsic* parameters of a camera.

$$\mathbf{x} = KR[I_3 | -\mathbf{X}_o]\mathbf{X}$$

Zhang's Method

Compute the 5 *intrinsic* parameters in K.

$$\mathbf{x} = KR[I_3 | -\mathbf{X}_o]\mathbf{X}$$

Zhang's Method

Camera calibration using images of a **checkerboard**.

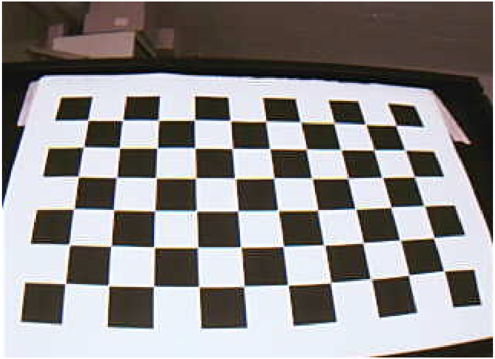

Figure 3: calibration target

Checkerboard

– Board is of **known** size and structure.
– The board must be **flat**.

Figure 4: Calibration targets

Checkerboard Method

Set the **world** coordinate system to the **corner** of the checkerboard.

- do this for *each* image captured.
- all points lie on x/y plane with z=0

Figure 5: Detected corners

Simplification

The Z coordinate of each point is **zero**.

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c & s & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Simplification

The last column of the rotation matrix has no effect on the system.

- we can delete these components from the system

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c & s & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & \color{red}{r_{13}} & t_1 \\ r_{21} & r_{22} & \color{red}{r_{23}} & t_2 \\ r_{31} & r_{32} & \color{red}{r_{33}} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ \color{red}{Z} \\ 1 \end{bmatrix}$$

Simplification

- The Z coordinate of each point is **zero**.
- Deleting the third column of R gives us:

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c & s & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Simplification

- Each observed point gives this equation.
- The *intrinsics* persist for **all** images.
- The *extrinsics* persist for **each** image.

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c & s & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Setting up the equations

Define a matrix H:

$$H = \left[\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3\right] = \begin{bmatrix} c & s & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix}$$

One point generates this equation:

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Setting up the equations

For multiple point observations:

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \underset{3\times 3}{H} \begin{bmatrix} X_i \\ Y_i \\ 1 \end{bmatrix}, \quad i = 1..., n$$

Analogous to the *DLT*.

Parameter Estimation

We estimate a 3×3 homography instead of 3×4 projection.

$$a_{x_i}^T \mathbf{h} = 0, \quad a_{y_i}^T \mathbf{h} = 0$$

with:

$$\begin{aligned} \mathbf{h} &= vec(H^T) \\ a_{x_i}^T &= (-X_i, -Y_i, -1, 0, 0, 0, x_i X_i, x_i Y_i, x_i) \\ a_{y_i}^T &= (0, 0, 0, -X_i, -Y_i, -1, y_i X_i, y_i Y_i, y_i) \end{aligned}$$

Parameter Estimation

Solving the system of linear equations leads to an estimate of the parameters of H.

- We need to identify **at least** 4 points.
- H has 8 Dof (degrees of freedom)
- each point provides 2 observations

We now have the parameters of H, how do we find K?

Decompose Intrinsic Parameters

For the DLT, we could use QR decomposition to find the rotation matrix of the extrinsic parameters.

- We can not do this for Zhang's method.
- We eliminated part of R earlier.

Decompose Intrinsic Parameters

$$H = \left[\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3\right] = \underbrace{\begin{bmatrix} c & s & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix}}_{K} \underbrace{\begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix}}_{[\mathbf{r}_1, \mathbf{r}_2, \mathbf{t}]}$$

Decompose Intrinsic Parameters

We need to extract K from the matrix $H = K[\mathbf{r}_1, \mathbf{r}_2, \mathbf{t}]$ we computed using SVD.

Decompose Intrinsic Parameters

We need to extract K from the matrix $H = K[\mathbf{r}_1, \mathbf{r}_2, \mathbf{t}]$ we computed using SVD.

Four step process:

1. Exploit constraints of $K, \mathbf{r}_1, \mathbf{r}_2$
2. Define a matrix $B = K^{-T}K^{-1}$
3. Solve B using another homogeneous linear system.
4. Decompose B.

Exploiting Constraints

What constraints do we have?

Exploiting Constraints

$$K = \begin{bmatrix} c & s & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix}$$

K is **invertible**.

Exploiting Constraints

$$H = \left[\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3\right] = \underbrace{\begin{bmatrix} c & s & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix}}_{K} \underbrace{\begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix}}_{[\mathbf{r}_1, \mathbf{r}_2, \mathbf{t}]}$$

$$[\mathbf{r}_1, \mathbf{r}_2, \mathbf{t}] = K^{-1}[\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3]$$

$$\Rightarrow \mathbf{r}_1 = K^{-1}\mathbf{h}_1, \quad \mathbf{r}_2 = K^{-1}\mathbf{h}_2$$

Exploiting Constraints

As $[\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3]$ are the columns of a rotation matrix, they form an orthonormal basis.

$$\mathbf{r}_1^T \mathbf{r}_2 = 0, \quad ||\mathbf{r}_1|| = ||\mathbf{r}_2|| = 1$$

Exploiting Constraints

$$\mathbf{r}_1 = K^{-1}\mathbf{h}_1, \quad \mathbf{r}_2 = K^{-1}\mathbf{h}_2, \quad \mathbf{r}_1^T\mathbf{r}_2 = 0, \quad ||\mathbf{r}_1|| = ||\mathbf{r}_2|| = 1$$

$$\mathbf{h}_1^T K^{-T} K^{-1}\mathbf{h}_2 = 0$$

$$\mathbf{h}_1^T K^{-T} K^{-1}\mathbf{h}_1 = \mathbf{h}_2^T K^{-T} K^{-1}\mathbf{h}_2$$

$$\mathbf{h}_1^T K^{-T} K^{-1}\mathbf{h}_1 - \mathbf{h}_2^T K^{-T} K^{-1}\mathbf{h}_2 = 0$$

Exploiting Constraints

$$\mathbf{h}_1^T K^{-T} K^{-1} \mathbf{h}_2 = 0$$

$$\mathbf{h}_1^T K^{-T} K^{-1} \mathbf{h}_1 - \mathbf{h}_2^T K^{-T} K^{-1} \mathbf{h}_2 = 0$$

Exploiting Constraints

Define a matrix $B := K^{-T} K^{-1}$

$$\mathbf{h}_1^T B \mathbf{h}_2 = 0$$

$$\mathbf{h}_1^T B \mathbf{h}_1 - \mathbf{h}_2^T B \mathbf{h}_2 = 0$$

Exploiting Constraints

From B the calibration matrix can be recovered using *Cholesky* decomposition.

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$chol(B) = AA^T \Rightarrow A = K^{-T}$$

If we know B, we can recover the calibration matrix K.

Exploiting Constraints

What do we have so far?

$$\mathbf{h}_1^T B \mathbf{h}_2 = 0$$

$$\mathbf{h}_1^T B \mathbf{h}_1 - \mathbf{h}_2^T B \mathbf{h}_2 = 0$$

- Matrix B, which is symmetric positive, so 6 unknowns.
- $\mathbf{h}$ are known.
- Two equations that relate B and $\mathbf{h}$.

Exploiting Constraints

Define a vector $\mathbf{b} = (b_{11}, b_{12}, b_{13}, b_{22}, b_{23}, b_{33})$

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{bmatrix}$$

There are 6 unknowns in B, because it is symmetric.

Exploiting Constraints

Construct a system of equations $V\mathbf{b} = 0$ exploiting our constraints.

$$v_{12}^T\mathbf{b} = 0, \quad v_{11}^T\mathbf{b} - v_{22}^T\mathbf{b} = 0$$

Matrix V

The matrix V is given by:

$$V = \begin{bmatrix} v_{12}^T \\ v_{11}^T - v_{22}^T \end{bmatrix}, \quad with \quad v_{ij} = \begin{bmatrix} h_{1i}h_{1j} \\ h_{1i}h_{2j} + h_{2i}h_{1j} \\ h_{3i}h_{1j} + h_{1i}h_{3j} \\ h_{2i}h_{2j} \\ h_{3i}h_{2j} + h_{2i}h_{3j} \\ h_{3i}h_{3j} \end{bmatrix}$$

Matrix V

For **each** image we get:

$$\begin{bmatrix} v_{12}^T \\ v_{11}^T - v_{22}^T \end{bmatrix} \mathbf{b} = 0$$

Matrix V

For multiple images we stack the matrices to a $2n \times 6$ matrix:

$$\begin{bmatrix} v_{12}^T \\ v_{11}^T - v_{22}^T \\ \dots \\ v_{12}^T \\ v_{11}^T - v_{22}^T \end{bmatrix} \mathbf{b} = 0$$

We need to solve the linear system of $V\mathbf{b} = 0$ to find b and hence K.

Solving the Linear System

The system $V\mathbf{b} = 0$ has a trivial solution when $\mathbf{b} = 0$ which will not provide a valid matrix B.

- Apply additional constraint $||\mathbf{b}|| = 1$.

Solving the Linear System

Real world measurements are noisy.

– Find the solution that minimises the least squares error:

$$b^* = arg\,\min_b ||V\mathbf{b}|| \quad \text{with} \quad ||\mathbf{b}|| = 1$$

Use SVD and choose the singular vector corresponding to the smallest singular value.

Minimum Requirements

- At least 4 points in each target image.
- Each target image gives *two* equations.
- B has 6 DoF so we need 3 *different* views of the target.
- Solve $V\mathbf{b} = 0$ using SVD to compute K.

Non-Linear Distortion

How to deal with non-linear distortion?

Non-Linear Distortion

A general calibration matrix is obtained by multiplying the affine camera with a general mapping.

$$ {}^{a}H(\mathbf{x}, q)K = \begin{bmatrix} 1 & 0 & x\Delta(\mathbf{x}, q) \\ 0 & 1 & y\Delta(\mathbf{x}, q) \\ 0 & 0 & 1 \end{bmatrix} $$

Lens Distortion

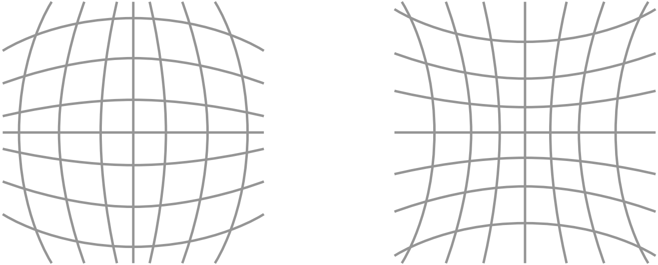

Figure 6: barrel and pincushion distortion

Radial Distortion

A standard approach for radial distortion:

$$
\begin{aligned}
{}^{a}x &= x(1 + q_1 r^2 + q_2 r^4) \\
{}^{a}y &= y(1 + q_1 r^2 + q_2 r^4)
\end{aligned}
$$

- with $[x, y]^T$ a point projected by the ideal camera.
- with r the distance from the camera principal point to the pixel.
- q_1 and q_2 are the radial distortion parameters.

Lens Distortion

Lens distortion can be calculated by minimising a non-linear function.

- Make an initial guess for the distortion parameters.
- Calculate K using Zhang's method.
- Measure the reprojection error.
- Refine the distortion parameters.

Figure 7: before calibration

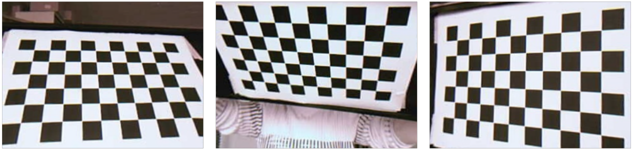

Figure 8: after calibration

Packages

These, and many other methods for calibration, are available in popular image processing packages.

- OpenCV for python and C++.
- Camera Calibration Toolkit for Matlab.

Summary

- Pinhole camera model.
- Non-linear model for distortion.
- Calibration using images of a target.

Reading:

- Forsyth, Ponce; Computer Vision: A modern approach.
- Hartley, Zisserman; Multiple View Geometry in Computer Vision
- Zhang, Z., A flexible new technique for camera calibration.