Epipolar Geometry

Computer Vision CMP-6035B

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Contents

Motivation

Given x' in the first image, **find** the corresponding point x'' in the second image.

- Coplanarity constraint
- Intersection of two corresponding rays
- The rays lie in a 3D plane

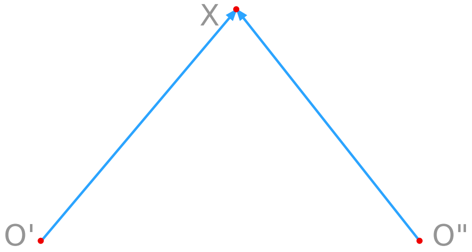

Figure 1: Coplanarity

Epipolar Geometry

- describe *geometric* relations in image pairs
- *efficient* search and prediction of corresponding points
- search space reduced from 2D to 1D

Epipolar Geometry

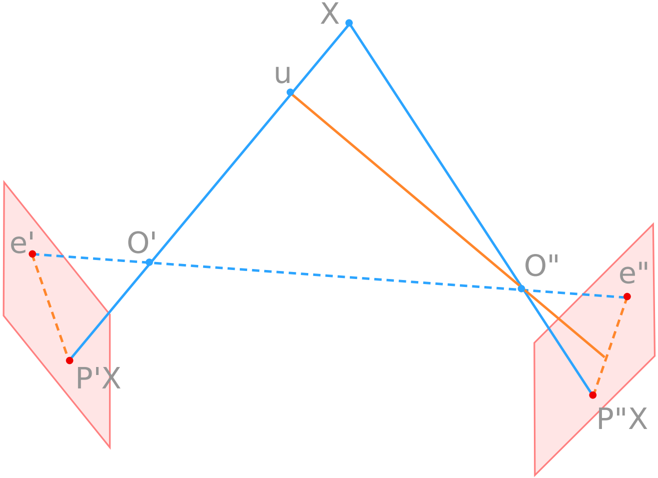

Figure 2: Epipolar Geometry

Epipolar Geometry

Epipolar elements:

- **epipolar axis** $\mathcal{B} = (O'O'')$
- **epipolar plane** $\mathcal{E} = (O'O''X)$
- **epipoles** $e' = (O'')', e'' = (O')''$
- **epipolar lines** $\mathcal{L}'(X) = (O''X)', \mathcal{L}''(X) = (O'X)''$

We can also write the **epipoles** as:

$$e' = (O'O'') \cap \mathcal{E}', \quad e'' = (O'O'') \cap \mathcal{E}''$$

And the **epipolar lines** as:

$$\mathcal{L}'(X) = \mathcal{E} \cap \mathcal{E}', \quad \mathcal{L}''(X) = \mathcal{E} \cap \mathcal{E}''$$

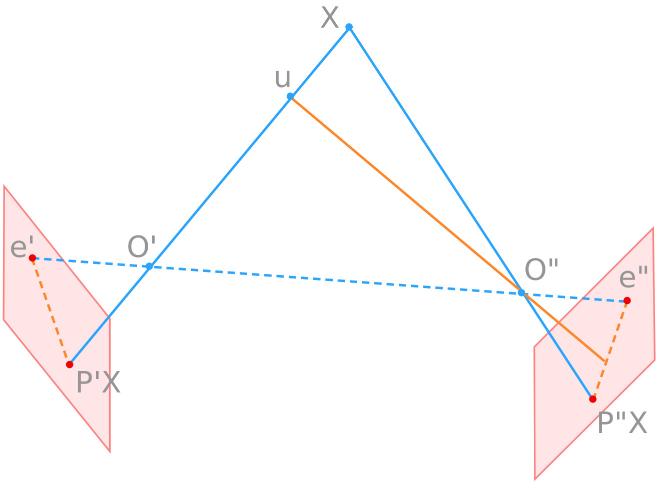

Figure 3: Epipolar Geometry

In the Epipolar Plane

Assuming a distortion free lens:

- The projection centres O' and O''.
- The observed point X.
- The epipolar lines, $\mathcal{L}'(X)$ and $\mathcal{L}''(X)$.
- The epipoles, e' and e''.
- The image points x' and x''.

In the Epipolar Plane

- The projection centres O' and O''.
- The observed point X.
- The epipolar lines, $\mathcal{L}'(X)$ and $\mathcal{L}''(X)$.
- The epipoles, e' and e''.
- The image points x' and x''.

All lie in the epipolar plane $\mathcal{E}$.

Predicting Point Correspondence

Task: Predict the location of x'' given x'.

- For the epipolar plane $\mathcal{E} = (O'O''X)$
- The intersection of $\mathcal{E}$ and the second image plane $\mathcal{E}''$ yields the epipolar line $\mathcal{L}''(X)$
- The corresponding point x'' lies on that epipolar line $\mathcal{L}''(X)$.
- Search space is *reduced* from 2D to 1D.

Computing the Elements of Epipolar Geometry

Computing the Elements of Epipolar Geometry

- We described the important elements geometrically.
- We will *compute* them using the **projection** matrices and the **fundamental** matrix.

Epipolar Axis

The direction of the epipolar axis can be derived directly from the projection centres.

$$b = X_{O'} - X_{O''}$$

The vector b is *homogeneous*, we know only the *direction*, **not** the *length*.

Epipolar Lines

Image points lie on the epipolar lines.

$$x' \in \mathcal{L}', \quad x'' \in \mathcal{L}''$$

Epipolar Lines

For x':

$$x'^T \mathcal{L}' = 0$$

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We can exploit the coplanarity constraint for x' and x'':

$$x'^T \underbrace{F x''}_{\mathcal{L}'} = 0$$

Epipolar Lines

We can exploit the coplanarity constraint for x' and x'':

$$x^{'T} \underbrace{Fx''}_{\mathcal{L}'} = 0$$

$$\mathcal{L}' = Fx''$$

Epipolar Lines

The same for x'':

$$\mathcal{L}''^T x'' = 0$$

We can exploit the same constraint $x'^T F x'' = 0$ and obtain:

$$\mathcal{L}''^T = x'^T F$$

Epipolar Lines

We can exploit the same constraint $x'^T F x'' = 0$ and obtain:

$$\mathcal{L}''^T = x'^T F$$

$$\mathcal{L}'' = F^T x'$$

Epipolar Lines

Image points lie on the epipolar lines, $x' \in \mathcal{L}'$ and $x'' \in \mathcal{L}''$.

- we can exploit the coplanarity constraint $x'^T F x'' = 0$.
- which is valid if:

$$\mathcal{L}' = Fx'' \quad \mathcal{L}'' = F^T x'$$

Epipoles

The epipoles are the projection of the camera origin onto the other image.

- Both can be computed using the projection matrices.

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The epipoles are the projection of the camera origin onto the other image.

– Both can be computed using the projection matrices.

$$e' = P' X_{O''} \quad e'' = P'' X_{O'}$$

Epipoles

The epipole is the *intersection* of **all** the epipolar lines in an image.

$$\forall \mathcal{L}' : e'^{T} \mathcal{L}' = 0 \quad \forall \mathcal{L}'' : e''^{T} \mathcal{L}'' = 0$$

Summary

- We assumed an uncalibrated camera.
- We discussed Epipolar geometry and epipolar elements.
- Epipolar geometry reduces the correspondence search from 2D to 1D.

Reading:

- Forsyth, Ponce; Computer Vision: A modern approach.
- Hartley, Zisserman; Multiple View Geometry in Computer Vision.

Data:

- Middlebury Stereo Datasets
- ETH Zurich 3D