Representing Shapes

Computer Vision CMP-6035B

Dr. David Greenwood

david.greenwood@uea.ac.uk

SCI 2.16a University of East Anglia

February 5, 2022

Content

- Chain codes
- Elliptical Fourier Descriptors

Shapes

Shapes compactly describe objects in images.

Representing Shapes

A shape in an image could be represented using the coordinates of edge pixels.

Representing Shapes

Pixel coordinates encode the *shape* **and** the *location*

- describes the shape in the image coordinate frame
- same shape in two locations appears to be different

Representing Shapes

We are not interested in where the shape is - just the representation of the shape itself.

Chain Codes

Rather than represent edge pixels in terms of image coordinates, represent each pixel as a **direction**.

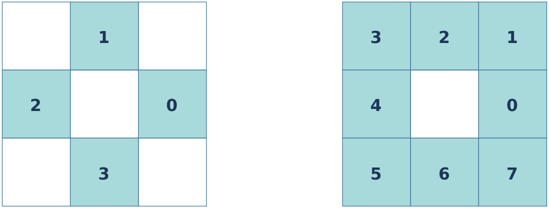
Chain Codes

In which direction must we move to stay on the edge?

- Shape is a *sequence of directions*.
- This is a **chain code**.

Connectivity

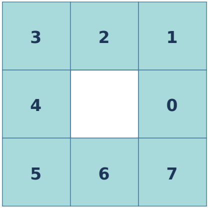
- Connectivity is the notion of pixels being connected.
- A path must pass through connected pixels.
- In which directions can we travel to stay on the path?



	1	
2		0
	3	

3	2	1
4		0
5	6	7

Figure 1: 4 and 8 connectivity



3	2	1
4		0
5	6	7

Figure 2: We will use 8 connectivity

Chain Code Example

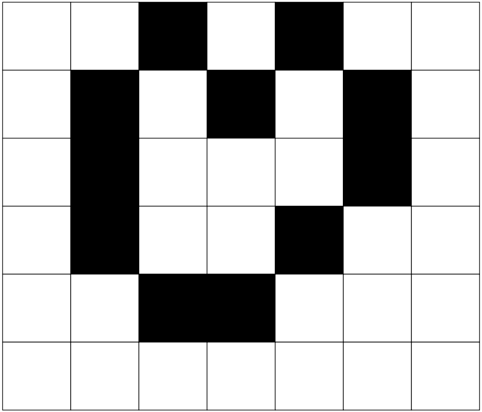


Figure 3: Encode this image

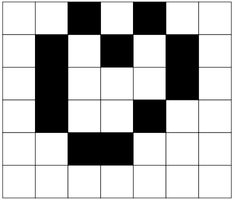


Figure 4: Encoding assumptions

Assume:

- 8 connectivity
- scan anti-clockwise
- start at left-most column, then top-most row
- edge pixels are black

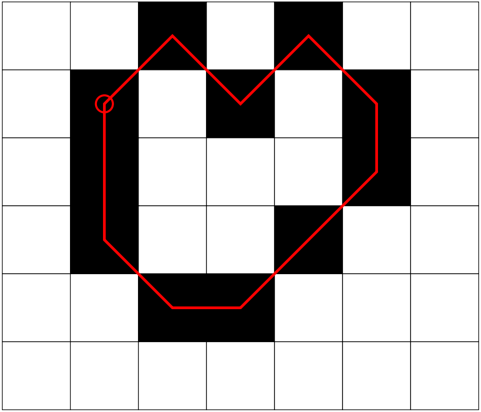


Figure 5: The edge boundary

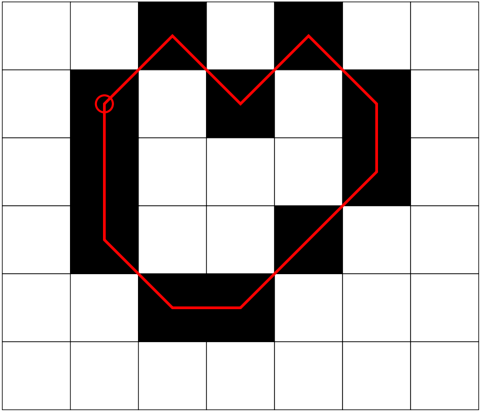


Figure 6: Resulting code: 6 6 7 0 1 1 2 3 5 3 5

Chain Codes

66701123535

Chain Codes

For invariance to starting location:

- compute the chain code and rotate so the code represents the smallest m-digit **shape-number**.
- 66701123535 → 01123535667

Chain Codes

Chain codes *are* **translation** invariant.

- Adding a constant value to the x, y coordinates does not change the shape.

Chain codes are **not** scale or rotation invariant.

Chain Code Derivatives

Chain codes specify a direction in absolute terms.

- Eg. 0 represents East, regardless of current direction.

Chain Code Derivatives

This idea can be extended to use a relative encoding.

- Represent the next direction as the number of turns required to stay on the shape boundary.
- In this case, 0 corresponds to straightforward.
- This is a chain code *derivative* or differential chain code.

Chain Code Derivatives

To compute the chain code derivative:

- Compute the difference between chain code elements.
- Take the result *modulo* n (the connectivity).

Chain Code Derivatives

Need to be careful with the starting element.

- Common assumption is begin straightforward.
- Chain code wraps around, so starting code is relative to the last.

Chain Code Derivatives

- Chain Code: 66701123535
- Derivative: 10111011262

NB: pay attention to modulus of negative numbers.

Chain Code Derivatives

Chain code derivative provides *rotational* invariance for rotations of **90 degrees**.

Chain Code Advantages

- compact representation - only boundary is stored
- invariant to translation
- easy to compute shape related features, e.g. area, perimeter, centroid

Chain Code Disadvantages

- No true rotational invariance and no scale invariance.
- Extremely sensitive to noise, sub-sampling loses definition.
- Cannot have sub-pixel accurate descriptions, only 4 or 8-connectivity.

Chain Code Disadvantages

Chain codes describe a specific instance of a shape.

- What about a class of non-rigid shapes?
- What about boundaries that are not closed?
- What about locating shapes automatically in images?

Elliptical Fourier Descriptors

A **parametric** representation of a shape.

Aside: Fourier Series

A Fourier series is an expansion of a **periodic** function $f(x)$ in terms of an infinite **sum** of sines and cosines.

Aside: Fourier Series

We can approximate non-periodic functions on a specific *interval*.

- by pretending the non-periodic part *is* periodic **outside** the interval.

Aside: Fourier Series

The Fourier series of a periodic function $f(t)$ of period T is:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos \frac{2\pi nt}{T} + b_n \sin \frac{2\pi nt}{T} \right]$$

for some set of Fourier coefficients a_n and b_n defined by the integrals:

$$a_n = \frac{2}{T} \int_0^T f(t) \cos \frac{2\pi nt}{T} \mathrm{d}t, \;\; b_n = \frac{2}{T} \int_0^T f(t) \sin \frac{2\pi nt}{T} \mathrm{d}t.$$

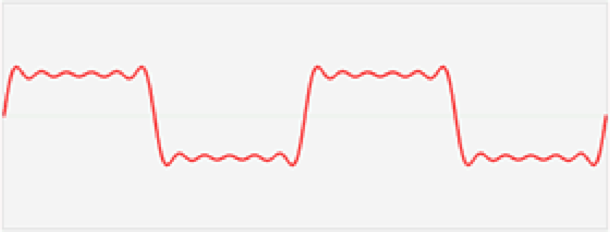


Figure 7: approximate square wave - Creative Commons

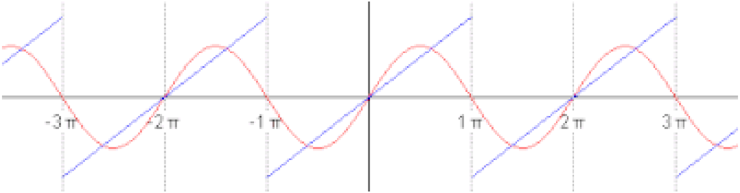


Figure 8: approximate saw tooth wave - public domain

Aside: Fourier Series

A function is **even** when:

$$f(x) = f(-x) \text{ for all } x$$

It has *reflective* symmetry about the **y-axis**, e.g. x^2 or $cos(x)$.

We can approximate even functions using only *cosine* coefficients.

Aside: Fourier Series

A function is **odd** when:

$$-f(x) = f(-x) \text{ for all } x$$

It *rotational* symmetry about the **origin**, e.g. x^3 or $sin(x)$.

We can approximate even functions using only *sine* coefficients.

It is useful to know about odd and even functions, but generally we will need to know both coefficients.

Elliptical Fourier Series

How do we go from Chain encodings to EFDs?

- First *separate* chain encodings into x and y **projections**.
- Allows us to deal with each dimension independently.

The projection of the first p links is the sum of differences between all previous links.

$$x_p = \sum_{i-1}^{p} \Delta x_i, \; y_p = \sum_{i-1}^{p} \Delta y_i$$

For the x-projection:

- For East, North East, or South East, $\Delta x = 1$.
- For North and South, $\Delta x = 0$.
- For West, North West, or South West, $\Delta x = -1$.

Similarly, for the y-projection:

- For North, North East, or North West, $\Delta y = 1$.
- For East and West, $\Delta y = 0$.
- For South, South East, or South West, $\Delta y = -1$.

We will consider the **“time”** derivative of the chain.

Time here means the *length* of the chain.

- The contribution of horizontal and vertical links is one.
- The contribution of a diagonal link is $\sqrt{2}$.

$$t_p = \sum_{i-1}^{p} \Delta t_i$$

Elliptical Fourier Series

Calculate the Fourier expansion for the x-projection.

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos \frac{2\pi nt}{T} + b_n \sin \frac{2\pi nt}{T} \right]$$

NB: not to infinity, but to some useful number of coefficients.

where:

$$\frac{a_0}{2} = \frac{1}{T}\int_0^T x(t)\ \mathrm{d}t$$

and T is the length of the chain.

again, from the definition:

$$a_n = \frac{2}{T}\int_0^T x(t)\cos\frac{2\pi nt}{T}\ \mathrm{d}t,\ \ b_n = \frac{2}{T}\int_0^T x(t)\sin\frac{2\pi nt}{T}\ \mathrm{d}t.$$

How can we calculate these coefficients?

The time derivative of x is periodic with period T and can itself be represented by the Fourier series:

$$x'(t) = \sum_{n=1}^{\infty} \alpha_n \cos \frac{2\pi nt}{T} + \beta_n \sin \frac{2\pi nt}{T}$$

where:

$$\alpha_n = \frac{2}{T} \int_0^T x'(t) \cos \frac{2\pi nt}{T} \mathrm{d}t \ , \beta_n = \frac{2}{T} \int_0^T x'(t) \sin \frac{2\pi nt}{T} \mathrm{d}t$$

then:

$$\alpha_n = \frac{2}{T} \int_0^T x'(t) \cos \frac{2\pi n t}{T} \mathrm{d}t$$

The difference here is our chain code is a piecewise linear function, so the time derivative is constant.

$$\begin{aligned}\alpha_n &= \frac{2}{T}\int_0^T x'(t)\cos\frac{2\pi nt}{T}\mathrm{d}t \\ &= \frac{2}{T}\sum_{p=1}^{K}\frac{\Delta x_p}{\Delta t_p}\int_{t_{p-1}}^{t_p}\cos\frac{2\pi nt}{T}\mathrm{d}t\end{aligned}$$

The “trick” is to notice that the integral over the whole period is a summation of the K chain links, and the derivative is a constant: the change in direction over the change in length.

finally, we take the antiderivative of the cosine term:

$$\begin{aligned}
\alpha_n &= \frac{2}{T}\int_0^T x'(t)\cos\frac{2\pi nt}{T}\mathrm{d}t \\
&= \frac{2}{T}\sum_{p=1}^{K}\frac{\Delta x_p}{\Delta t_p}\int_{t_{p-1}}^{t_p}\cos\frac{2\pi nt}{T}\mathrm{d}t \\
&= \frac{1}{n\pi}\sum_{p=1}^{K}\frac{\Delta x_p}{\Delta t_p}\left(\sin\frac{2\pi nt_p}{T} - \sin\frac{2\pi nt_{p-1}}{T}\right)
\end{aligned}$$

similarly, we can calculate:

$$\beta_n = \frac{1}{n\pi} \sum_{p=1}^{K} \frac{\Delta x_p}{\Delta t_p} \left(\cos \frac{2\pi n t_p}{T} - \cos \frac{2\pi n t_{p-1}}{T} \right)$$

We can also obtain $x'(t)$ directly from the $x(t)$ definition:

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{2\pi nt}{T} + b_n \sin \frac{2\pi nt}{T}$$

$$x'(t) = \sum_{n=1}^{\infty} -\frac{2\pi nt}{T} a_n \sin \frac{2\pi nt}{T} + \frac{2\pi nt}{T} b_n \cos \frac{2\pi nt}{T}$$

If we compare both derivations of $x'(t)$:

$$x'(t) = \sum_{n=1}^{\infty} \alpha_n \cos \frac{2\pi nt}{T} + \beta_n \sin \frac{2\pi nt}{T}$$

$$x'(t) = \sum_{n=1}^{\infty} -\frac{2\pi nt}{T} a_n \sin \frac{2\pi nt}{T} + \frac{2\pi nt}{T} b_n \cos \frac{2\pi nt}{T}$$

we can equate coefficients from both equations:

$$-\frac{2\pi nt}{T} a_n = \beta_n, \;\; \frac{2\pi nt}{T} b_n = \alpha_n$$

and solve for a_n and b_n yielding the x projection coefficients:

$$a_n = \frac{T}{2n^2\pi^2} \sum_{p=1}^{K} \frac{\Delta x_p}{\Delta t_p} \left(\cos \frac{2\pi n t_p}{T} - \cos \frac{2\pi n t_{p-1}}{T} \right)$$

$$b_n = \frac{T}{2n^2\pi^2} \sum_{p=1}^{K} \frac{\Delta x_p}{\Delta t_p} \left(\sin \frac{2\pi n t_p}{T} - \sin \frac{2\pi n t_{p-1}}{T} \right)$$

we can also solve for the y projection in the same way:

$$c_n = \frac{T}{2n^2\pi^2} \sum_{p=1}^{K} \frac{\Delta y_p}{\Delta t_p} \left(\cos \frac{2\pi n t_p}{T} - \cos \frac{2\pi n t_{p-1}}{T} \right)$$

$$d_n = \frac{T}{2n^2\pi^2} \sum_{p=1}^{K} \frac{\Delta y_p}{\Delta t_p} \left(\sin \frac{2\pi n t_p}{T} - \sin \frac{2\pi n t_{p-1}}{T} \right)$$

We now know everything we need to calculate the Fourier series coefficients for the x and y projections.

- The number of harmonics is n.
- The length of the chain is T.
- The number of chain links is K.
- The length of each link is t_p.

Elliptical Fourier Series

The DC component determines the centre position of the ellipse.

For those interested, the calculation can be found here:

“Kuhl, Giardina; Elliptic Fourier Features of a Closed Contour, Computer Graphics and Image Processing, 1982”

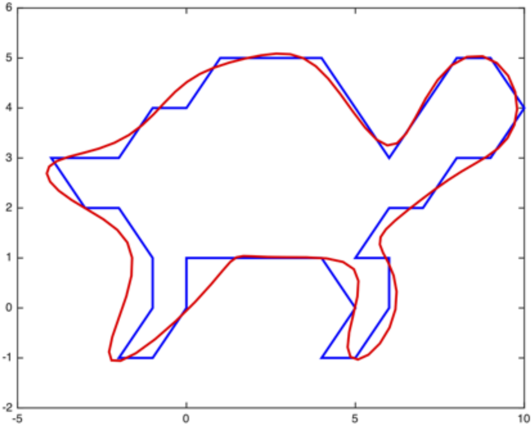


Figure 9: Elliptical Cat

Summary

Chain Codes

- conceptually simple
- affected by noise
- only really translation invariant

Elliptical Fourier Descriptors (EFDs)

- invariant to translation, scale and rotation
- less affected by noise
- very compact with fewer harmonics