Two-View Geometry

Computer Vision CMP-6035B

Dr. David Greenwood

david.greenwood@uea.ac.uk

SCI 2.16a University of East Anglia

Spring 2022

Contents

Camera Pair

Two cameras capturing images of the same scene.

Camera Pair

Figure 1: A stereo camera. Intel D435

Camera Pair

- A stereo camera.
- Two cameras, each with a different position.
- One camera that moves.

A **camera pair** is two configurations from which images have been taken of the same scene.

Orientation

The **orientation** of the camera pair can be described using *independent* orientations for each camera.

How many parameters are needed?

Orientation

The **orientation** of the camera pair can be described using *independent* orientations for each camera.

How many parameters are needed?

- *Calibrated* cameras require **12** parameters.
- *Uncalibrated* cameras require **22** parameters.

Camera Motion

Can we **estimate** the camera motion without *knowing* the scene?

Camera Motion

Which parameters can be obtained from these images?

- and which cannot?

Cameras Measure Direction

We can't obtain *global* **translation** and **rotation** or **scale**.

Cameras Measure Direction

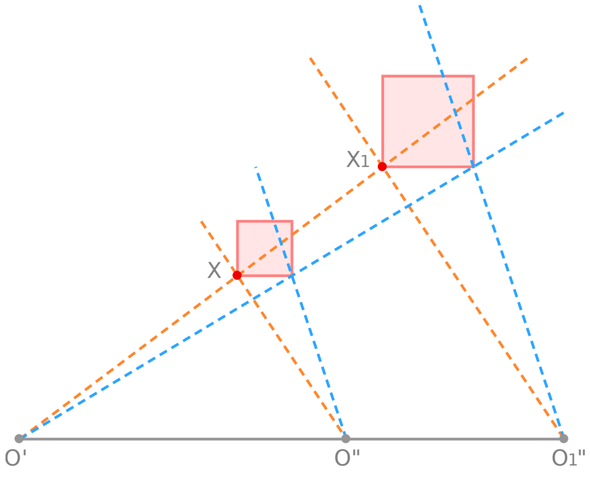

Figure 2: Two views

Cameras Measure Direction

We can obtain:

- 3 **rotation** parameters of the second camera *w.r.t.* the first camera.
- 2 **direction** parameters of the line B, connecting the two centres.
- But, we *can't* estimate the length of B.

Calibrated Cameras

- We need $2 \times 6 = 12$ parameters for two *calibrated* cameras for their pose.
- Without additional information we can only obtain $12 - 7 = 5$ parameters.
- Not 3 rotation, 3 translation, and 1 scale.

Photogrammetric Model

Given two cameras images, we *can* reconstruct an object up to a **similarity** transform.

Photogrammetric Model

The orientation of the photogrammetric model is called the **absolute** orientation.

- To *obtain* the absolute orientation we need at least 3 points in 3D.

Uncalibrated Cameras

For **uncalibrated** cameras, we can only obtain $22 - 15 = 7$ parameters given two images.

We need at **least 5 points** in 3D to obtain the absolute orientation.

Relative Orientation

Camera	image	pair	RO	AO	3D
Calibrated	6	12	5	7	3
Uncalibrated	11	22	7	15	5

- RO : relative orientation
- AO : absolute orientation
- 3D : minimum number of control points in 3D

Relative Orientation

By simply moving the camera in the scene we can obtain a **relative orientation**.

"Agarwal, Sameer, et al. Building rome in a day. 2011"

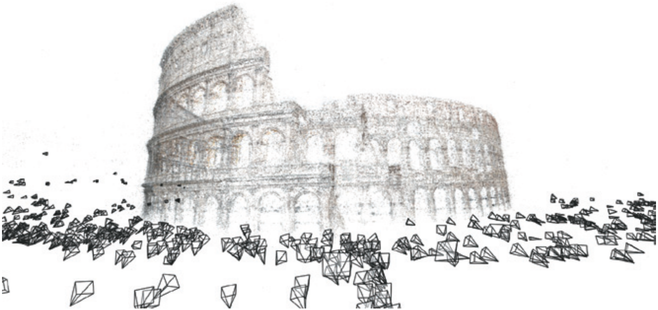

Figure 3: Rome in a day

Coplanarity Constraint

Leading to the Fundamental Matrix.

Coplanarity Constraint

Which parameters can we compute without any knowledge of the scene?

Coplanarity Constraint

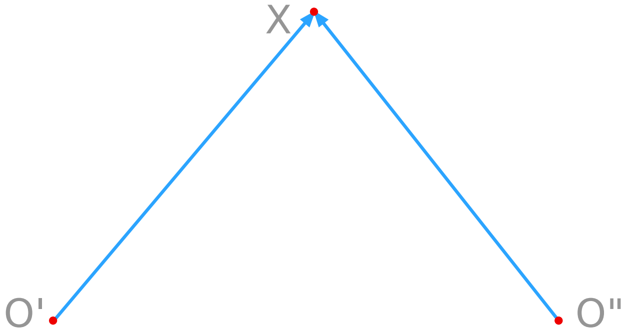

Figure 4: Two cameras observe one point.

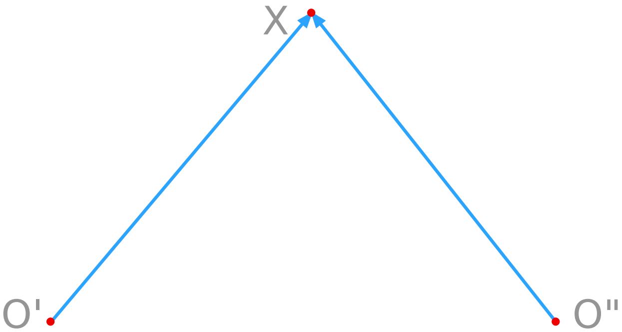

Figure 5: The perfect intersection of two rays.

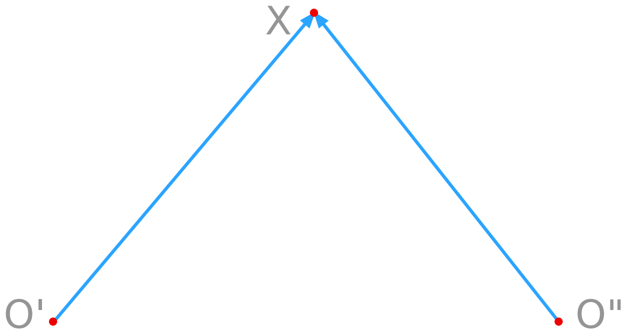

Figure 6: Two rays lie on a plane.

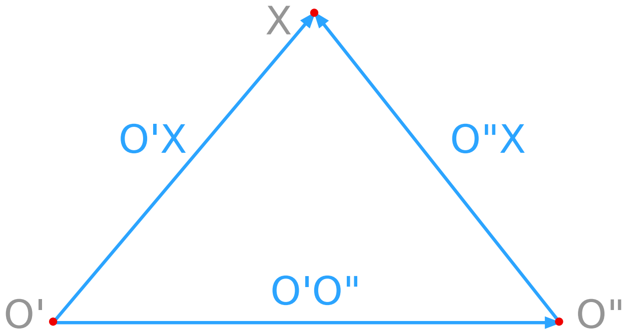

Figure 7: The baseline vector.

Coplanarity can be expressed in the following way:

$$[O^{'}X, O^{'}O^{''}, O^{''}X] = 0$$

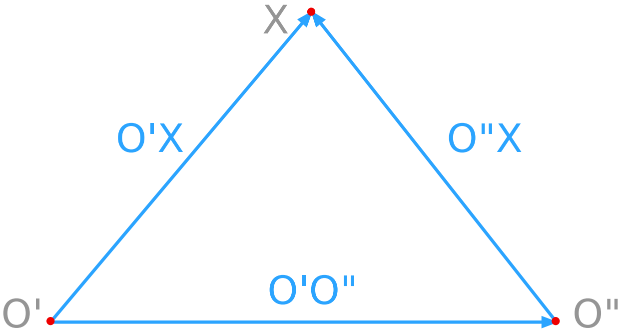

Figure 8: Coplanarity

Aside: Scalar Triple Product

Dot product of one vector with the cross product of the other two.

$$[A, B, C] = (A \times B) \cdot C$$

- It is the volume of the *parallelepiped* formed by the three vectors.
- $[A, B, C] = 0$ if all the vectors are in a **plane**.

Coplanarity

$$[O'X, O'O'', O''X] = 0$$

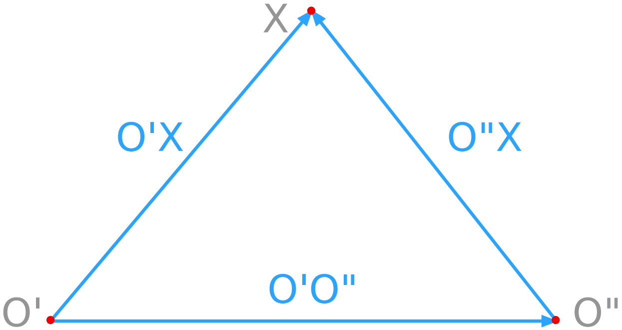

Figure 9: Coplanarity

Coplanarity for Uncalibrated Cameras

The directions of the vectors $O^{'}X$ and $O^{''}X$ can be derived from the image coordinates x', x'':

$$x' = P'X \qquad x'' = P''X$$

with the projection matrices:

$$P' = K'R'[\mathbf{I}_3| - X_{O'}] \qquad P'' = K''R''[\mathbf{I}_3| - X_{O''}]$$

Coplanarity for Uncalibrated Cameras

The normalised direction of the vector $O^{'}X$ is:

$$^{n}x^{'} = (R')^{-1}(K')^{-1}x'$$

Coplanarity for Uncalibrated Cameras

The *normalised* direction of the vector $O^{'}X$ is:

$$^{n}x^{'} = (R')^{-1}(K')^{-1}x'$$

as the *normalised* projection:

$$^{n}x^{'} = [\mathbf{I}_3 | -X_{O'}]X$$

This gives the **direction** from the centre of projection to the point in 3D.

Coplanarity for Uncalibrated Cameras

Analogously, we can do the same thing for both cameras:

$$ {}^{n}x' = (R')^{-1}(K')^{-1}x' \qquad {}^{n}x'' = (R'')^{-1}(K'')^{-1}x'' $$

Baseline Vector

The baseline vector $O^{'}O^{''}$, is obtained from the coordinates of the projection centres:

$$\mathbf{b} = X_{O''} - X_{O'}$$

Coplanarity Constraint

recall:

$$[O^{'}X, O^{'}O^{''}, O^{''}X] = 0$$

can be expressed as:

$$\begin{aligned}
\left[{}^{n}x^{'}, \mathbf{b}, {}^{n}x^{''}\right] &= 0 \\
{}^{n}x^{'} \cdot (\mathbf{b} \times {}^{n}x^{''}) &= 0 \\
{}^{n}x^{'T} S_b {}^{n}x^{''} &= 0
\end{aligned}$$

Skew Symmetric Matrix

How does this work?

$$
\begin{aligned}
{}^{n}x' \cdot (\mathbf{b} \times {}^{n}x'') &= 0 \\
{}^{n}x'^{T} S_b \, {}^{n}x'' &= 0
\end{aligned}
$$

Write the cross product as a skew symmetric matrix S_b:

$$
\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -b_3 x_2 & + & b_2 x_3 \\ b_3 x_1 & - & b_1 x_3 \\ -b_2 x_1 & + & b_1 x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -b_3 & b_2 \\ b_3 & 0 & -b_1 \\ -b_2 & b_1 & 0 \end{bmatrix}}_{S_b} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}
$$

Fundamental Matrix

We can continue to work with the coplanarity constraint, to build the **fundamental** matrix.

Fundamental Matrix

By combining ${}^{n}x' = (R')^{-1}(K')^{-1}x'$ and ${}^{n}x'^{T} S_b {}^{n}x'' = 0$

– we obtain:

$$x'^{T}(K')^{-T}(R')^{-T} S_b (R'')^{-1}(K'')^{-1}x'' = 0$$

Fundamental Matrix

By combining ${}^{n}x^{'} = (R')^{-1}(K')^{-1}x'$ and ${}^{n}x^{'T} S_b {}^{n}x^{''} = 0$

– we obtain:

$$x'^T \underbrace{(K')^{-T}(R')^{-T} S_b (R'')^{-1}(K'')^{-1}}_{F} x'' = 0$$

$$\begin{aligned} F &= (K')^{-T}(R')^{-T} S_b (R'')^{-1}(K'')^{-1} \\ &= (K')^{-T}(R') S_b (R'')^{T}(K'')^{-1} \end{aligned}$$

Fundamental Matrix

The matrix F is the **fundamental** matrix.

$$F = (K')^{-T}(R')S_b(R'')^T(K'')^{-1}$$

– it allows us to express the *coplanarity constraint* as:

$$x'^T F x'' = 0$$

Fundamental Matrix

The **fundamental matrix** holds the parameters we can estimate to describe the *relative orientation* of two cameras looking at the same point.

$$x'^T F x'' = 0$$

Fundamental Matrix

The **fundamental matrix** fulfils the equation:

$$x'^T F x'' = 0$$

for **corresponding points** in two images.

- The fundamental matrix contains **all** the *information about the relative orientation* of **two images** from uncalibrated cameras.

Fundamental Matrix

NOTE: we have defined the fundamental matrix for the relative orientation from camera one to camera two.

- You will also find in the literature, F can be defined for the relative orientation from camera two to camera one.
- This transposition must be accounted for when comparing expressions.

Essential Matrix

Calibrated Cameras

Calibrated Cameras

Most photogrammetric systems rely on calibrated cameras.

- Calibrated cameras *simplify* the orientation problem.
- Often, both cameras have the *same* calibration matrix.

Calibrated Cameras

For calibrated cameras the coplanarity constraint can be simplified.

– From the calibration matrices we obtain the directions as:

$$^{k}x' = (K')^{-1}x' \quad ^{k}x'' = (K'')^{-1}x''$$

Coplanarity

From the fundamental matrix:

$$x'^T F x'' = 0$$

$$x'^T \underbrace{(K')^{-T}(R')^{-T} S_b (R'')^{-1}(K'')^{-1}}_{F} x'' = 0$$

Coplanarity

From the fundamental matrix:

$$x'^T F x'' = 0$$

$$x'^T \underbrace{(K')^{-T}(R')^{-T} S_b (R'')^{-1}(K'')^{-1}}_{F} x'' = 0$$

$$\underbrace{x'^T (K')^{-T}}_{{}^k x'^T} (R')^{-T} S_b (R'')^{-1} \underbrace{(K'')^{-1} x''}_{{}^k x''} = 0$$

Coplanarity

From the fundamental matrix:

$$x'^T F x'' = 0$$

$$x'^T \underbrace{(K')^{-T}(R')^{-T} S_b (R'')^{-1}(K'')^{-1}}_{F} x'' = 0$$

$$\underbrace{x'^T (K')^{-T}}_{{}^k x'^T}(R')^{-T} S_b (R'')^{-1} \underbrace{(K'')^{-1} x''}_{{}^k x''} = 0$$

$${}^k x'^T \underbrace{R' S_b R''^T}_{E} {}^k x'' = 0$$

Essential Matrix

From F to the essential matrix E:

$$
{}^k\mathbf{x}'^{\mathsf{T}} \underbrace{R' S_b R''^{\mathsf{T}}}_{E} {}^k\mathbf{x}'' = 0
$$

$$
{}^k\mathbf{x}'^{\mathsf{T}} E\, {}^k\mathbf{x}'' = 0
$$

$$
E = R' S_b R''^{\mathsf{T}}
$$

Essential Matrix

The essential matrix is a *special form* of the fundamental matrix.

For **calibrated cameras** it is called the essential matrix:

$$E = R' S_b R''^T$$

For calibrated cameras, the *coplanarity constraint* is:

$$^k x'^T E \, ^k x'' = 0$$

Essential Matrix

- The essential matrix has **five** degrees of freedom.
- The essential matrix is *homogeneous* and *singular*.

$$ {}^k\mathbf{x}'^T E \, {}^k\mathbf{x}'' = 0 $$

Computing Relative Orientation

How do we obtain the values of the fundamental matrix from image correspondences?

8 Point algorithm

We know the direction vectors from the image coordinates, but the parameters of F are unknown.

$$[x'_n, y'_n, 1] \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} x''_n \\ y''_n \\ 1 \end{bmatrix} = 0$$

8 Point algorithm

Solve using the SVD:

$$A \begin{bmatrix} F_{11} \\ \vdots \\ F_{33} \end{bmatrix} = 0$$

8 Point algorithm

From **8** *corresponding* points, we can solve F or E.

Packages

There are implementations of these algorithms in many popular packages.

- OpenCV for python and C++.
- Camera Calibration Toolkit for Matlab.

Summary

- Camera Pair
- Coplanarity Constraint
- Fundamental Matrix
- Essential Matrix

Reading:

- Forsyth, Ponce; Computer Vision: A modern approach.
- Hartley, Zisserman; Multiple View Geometry in Computer Vision.
- H. Christopher Longuet-Higgins (1981). "A computer algorithm for reconstructing a scene from two projections".