Projections

Graphics 1 CMP-5010B

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Content

- The Camera Model
- Perspective Projection
- Orthographic Projection

Projection

From 3D to 2D...

Projection

To give a meaningful account of projection in graphics we have to move to 3D.

- Projection from 3D results in a 2D image.

Projection

There are typically two types of projections we consider in graphics:

- Perspective projection
- Orthographic projection

Camera Model

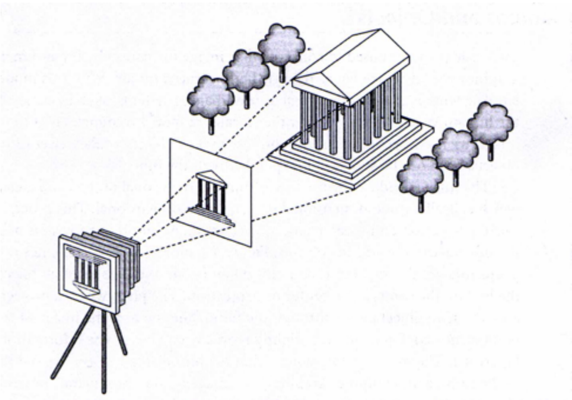

Figure 1: Camera Model: E. Angel, Interactive Computer Graphics

View Frustum

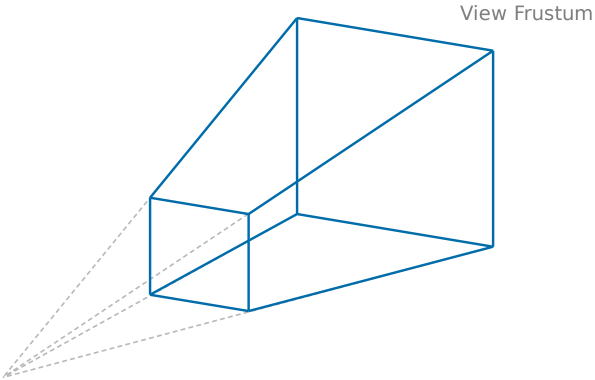

Figure 2: view frustum

View Frustum

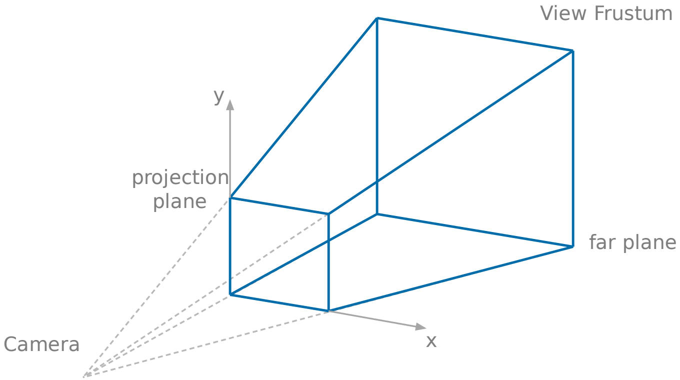

Figure 3: projection on near plane

View Frustum

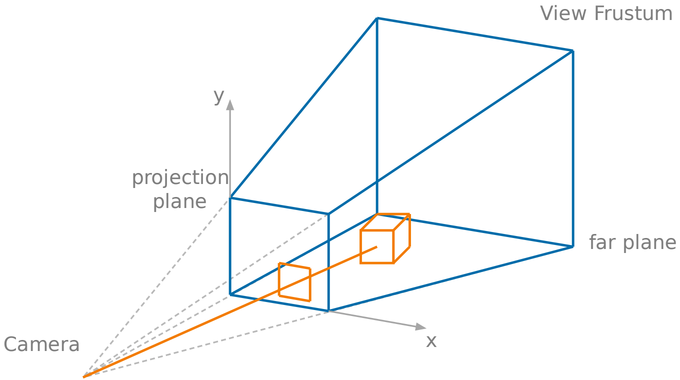

Figure 4: view frustum culling

View Frustum

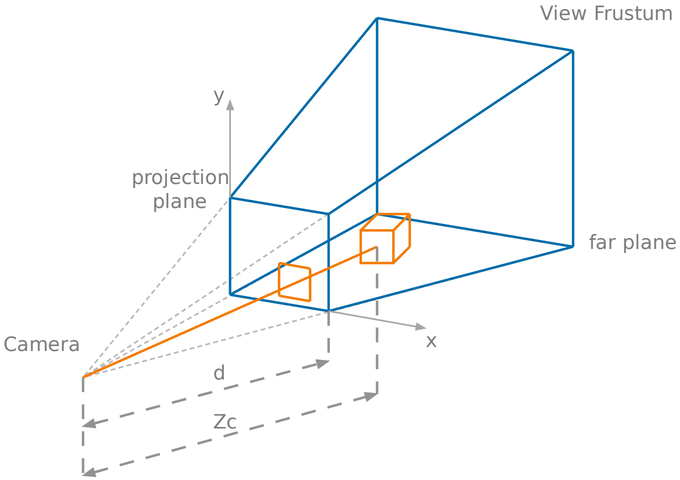

Figure 5: frustum metrics

Perspective Projection

Extend the idea of *homogeneous* coordinates to 3D.

Perspective Projection

The perspective projection is a projection from 3D to 2D, so we need a 4 x 4 matrix to transform 3D points in homogeneous coordinates.

consider a horizontal cross section of the scene:

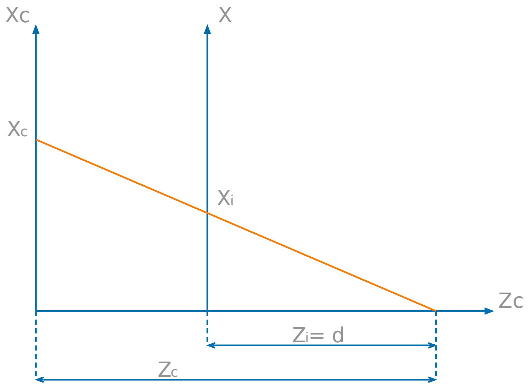

Figure 6: x-section

The relationship between the the 3D camera coordinate x_c and the 2D image coordinate x_i is:

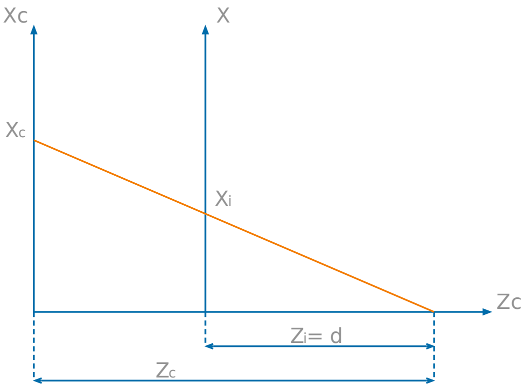

Figure 7: x-section

$$\frac{x_i}{d} = \frac{x_c}{z_c}$$

$$\Rightarrow x_i = \frac{x_c}{\frac{z_c}{d}}$$

The relationship between the the 3D camera coordinate y_c and the 2D image coordinate y_i is:

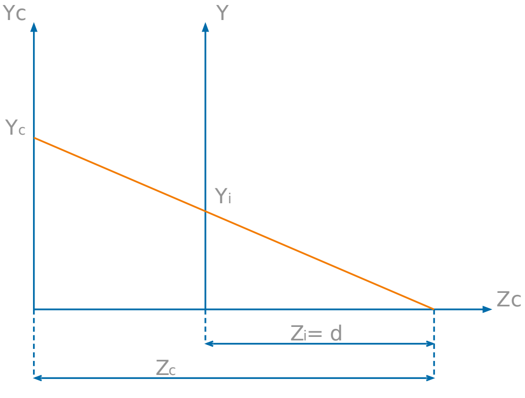

Figure 8: y-section

$$\frac{y_i}{d} = \frac{y_c}{z_c}$$

$$\Rightarrow y_i = \frac{y_c}{\frac{z_c}{d}}$$

Perspective Projection

We first extend the 3x3 homogeneous matrix for 2D graphics to a 4x4 matrix for 3D graphics.

A perspective projection from 3D to 2D can then be expressed as:

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Let's work out the matrix multiplication for each coordinate after projection:

$$x = x_c \ , \ y = y_c \ , \ z = z_c \ , \ w = \frac{z_c}{d}$$

- However, these are homogenous coordinates, i.e. they are scaled by a factor of w.
- take note that $w = \frac{z_c}{d}$

To find the corresponding image coordinates, we divide:

$$x_i = \frac{x}{w} = \frac{x_c}{\frac{z_c}{d}}\ , y_i = \frac{y}{w} = \frac{y_c}{\frac{z_c}{d}}\ , z_i = \frac{z}{w} = \frac{z_c}{\frac{z_c}{d}} = d,$$

so **all** z_i are equal to d.

Perspective Projection

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Orthographic Projection

parallel projection. . .

Orthographic Projection

Orthographic projection projects a 3D object to a 2D plane using parallel projection lines, perpendicular to the image plane.

Orthographic Projection

Parallel projection lines do *not* converge to a fixed camera point.

Orthographic Projection

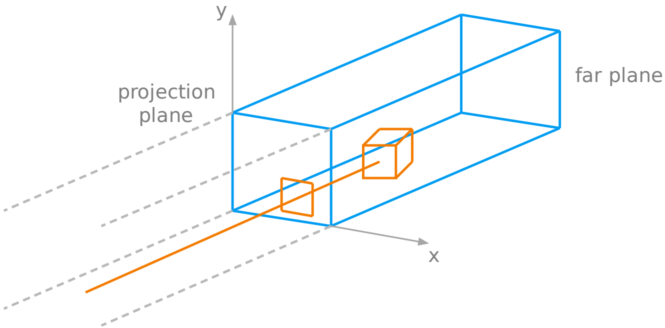

Figure 9: orthographic projection

Orthographic Projection

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Summary

- The Camera Model
- Perspective Projection
- Orthographic Projection

Reading:

- Hearn, D. et al. (2004). Computer Graphics with OpenGL.
- Strang, Gilbert, et al. (1993) Introduction to linear algebra.